are immediately found by forming the "square-matrix" for F (z) and so obtaining the conditions that F (z) should be a perfect square. The various relations so found connecting the quantities $p_1, p_2, \ldots, p_m$, and $m-1$ arbitrary quantities $\lambda_2, \ldots, \lambda_m$, are algebraic integrals of the above system of differential equations, and are all *rational* and *integral*. I then apply the general theorem to the case $m = 2$, or the case of elliptic integrals, and easily deduce the result given by Cayley in his work entitled an 'Elementary Treatise on Elliptic Functions' (p. 340).

I next apply the theory to the case of $m = 3$, and deduce two algebraic integrals, and show how the remaining relations may be found, and lastly to the case $m = 4$.

The next subject treated of is the source of F (z), from which we derive a differential equation which I call the *fundamental equation* in the theory of Abelian integrals and functions, as its integral leads us to a form which, when operated on by δ, leads us to a new algebraic equation, which again leads to another by a second application of the operator. By this method I obtain a number of interesting results, many of which are now given for the first time, as far as I am aware.

I then define Abelian functions and, by a method of treatment depending on what precedes, show that they are periodic functions and determine their periods.

We have at first sight $2m-1$ independent periods, and I reduce them to $2m-2$ by an easy application of the foregoing theory.

The above is a short abstract of what my paper contains, the most important portions of it being (a) the determination of the algebraic integrals in a *rational* and *integral* form; (b) the easy proof of the periodicity of Abelian functions.

I omit from this paper a discussion of the case in which the number of variables exceeds m, as likely to make my communication too lengthy.

III. "On the Application of the Kinetic Theory to Dense Gases" By S. H. Burbury, F.R.S. Received January 12, 1895.

(Abstract.)

1. Start with Clausius' equation

$$\tfrac{3}{2} p\mathrm{V} = \mathrm{T}_r + \tfrac{1}{2}\Sigma\Sigma\mathrm{R}r,$$

in which p denotes pressure per unit of area, V volume, and T_r kinetic energy of relative motion. Also R is the repulsive force, r the distance between the centres of two spheres, and the summation includes all pairs.

2. Evaluate $\Sigma\Sigma Rr$, on the assumption that no forces act except during collisions. That gives

$$\Sigma\Sigma Rr = \tfrac{2}{3}\pi c^3\rho \,.\, 2\rho T_r,$$

c being the diameter of a sphere and ρ the number of spheres in unit of volume.

Let $$\tfrac{2}{3}\pi c^3\rho = \kappa.$$

Then $$\Sigma\Sigma Rr = \kappa \,.\, 2\rho T_r,$$

and $$p = \tfrac{2}{3}(1+\kappa)\,\rho T_r.$$

3. This suggests that we should take for our law of distribution of energy, not ϵ^{-hT}, as in a rare medium, but $\epsilon^{-h(T+\kappa T_r)}$.

4. To test that suggestion, consider the case of an infinite vertical column of gas subject to a constant vertical force f. We have, if s be the height above a fixed horizontal plane, $dp/ds = -Mf\rho$.

Assuming for the moment the whole energy to be that of relative motion T_r, that gives

$$\tfrac{2}{3}\frac{d}{ds}(\overline{1+\kappa}\,\rho T_r) = -Mf\rho.$$

Now κ contains ρ as a factor. If we make T_r constant, as in the rare medium, the equation is impracticable. But make $\overline{1+\kappa}T_r$ constant $= 3/2h$, and we get the usual equation $\rho = \rho_0\epsilon^{-hMfs}$, ρ_0 being the value of ρ when $s = 0$.

5, 6, 7, 8. Now consider N spheres crossing the plane $s = 0$, with u for vertical component of velocity. Of these some will undergo collision before reaching ds. But an equal number will be substituted for them with the same vertical velocity, but with a small average advance in position in direction s, owing to the finite diameter c. It is shown that on average of the N spheres this advance is κds, and, therefore, the class of N spheres, original or substituted, will at the end of the time ds/u be at the height, not ds, but on average $(1+\kappa)ds$. But their loss of kinetic energy by the action of the force f is only $Mfds$. And, therefore, the loss due to the height ds is, allowing for substitutions, $Mfds/1+\kappa$.

9, 10. Hence we find that the assumption $\overline{1+\kappa}T_r = 3/2h$ satisfies all the conditions of equilibrium in exactly the same way as in the rare medium $T_r = 3/2h$ satisfies them.

11. The result can now be generalised by introducing stream motion, the energy of which is T_s, as well as that of relative motion T_r, and we find that $T+\kappa T_r$ must be constant throughout the column. Say, now, $T+\kappa T_r = 3/2h$.

12, 13. I have given elsewhere ('Science Progress,' November,

1894) reasons for assuming as the law of distribution of velocities among n spheres the expression

$$\epsilon^{-(a_1u_1^2+b_{12}u_1u_2+a_2u_2^2+b_{13}u_1u_3+b_{23}u_2u_3+\&c.)}$$

in which the coefficients have yet to be determined.

14, 15. The "a" coefficients must be all positive, the "b" coefficients all negative; and the b coefficients express the fact that the pairs of velocities to which they relate are not independent; and the b's, being negative, express the fact that the two velocities are more likely to be of the same than of opposite signs, so that there will be on the average of any group of contiguous spheres a greater common or stream motion than there would be were the velocities all independent.

16. The coefficients b must generally diminish as the distance between the two spheres to which they relate increases, becoming evanescent when that distance is great enough.

17. If the chance for a group of n spheres be of the form $C\epsilon^{-hQ_n}$, and for a group of $n-1$ spheres, part of the n spheres, $C\epsilon^{-hQ_{n-1}}$, Q_n and Q_{n-1} must be connected by the relation

$$\iiint_{-\infty}^{\infty} \epsilon^{-hQ_n} du_n dv_n dw_n = \epsilon^{-hQ_{n-1}}.$$

If we effect the integration for one variable we find, if

$$Q_n = a_1u_1^2+b_{12}u_1u_2+a_2u_2^2+\&c.$$

$$Q_{n-1} = a'_1u_1^2+b'_{12}u_1u_2+a'_2u_2^2+\&c.,$$

in which

$$2a'_1 = 2a_1 - \frac{b^2_{1n}}{2a_n};$$

$$b'_{12} = b_{12} - \frac{b_{1n}b_{2n}}{2a_n}; \ \&c.$$

This shows that as n diminishes the a coefficients diminish, and since every b coefficient is negative the b's increase in absolute value, so that the ratios bb'/a or b^2/a increase. On the other hand, as n increases the a's increase, and the squares and products of the form b^2 or bb' diminish. Whence it is inferred that as n increases the function

$$Q_n = a_1u_1^2+b_{12}u_1u_2+\&c.$$

tends to assume a limiting form. This limiting form must be T when $\kappa = 0$, and must be such as to make $\overline{T}_r$ less than it would be were all the velocities independent. It may then be assumed to be $T+\kappa T_r$.

18, 19. Assuming the law to be $\epsilon^{-h(\mathrm{T}+\kappa\mathrm{T}_r)}$, we have

$$\mathrm{T}+\kappa\mathrm{T}_r = \left(1+\frac{n-1}{n}\kappa\right)\frac{u_1^2}{2} - \frac{\kappa}{n}u_1u_2 - \frac{\kappa}{n}u_1u_3 - \&c.$$

$$+\left(1+\frac{n-1}{n}\kappa\right)\frac{u_2^2}{2} - \&c.,$$

and, forming the determinant,

$$\mathrm{D} = \left(1+\frac{n-1}{n}\kappa\right) - \frac{\kappa}{n} - \frac{\kappa}{n} -$$

$$-\frac{\kappa}{n}\left(1+\frac{n-1}{n}\kappa\right) - \frac{\kappa}{n} - \&c.$$

$$\cdots\cdots\cdots$$

With D_{11}, D_{12}, &c., for minors, we find

$$\mathrm{D} = (1+\kappa)^n - n\frac{\kappa}{n}(1+\kappa)^{n-1} = (1+\kappa)^{n-1}.$$

$$\mathrm{D}_{11} = \mathrm{D}_{22} = \&c. = (1+\kappa)^{n-1} - \overline{n-1}\,\frac{\kappa}{n}(1+\kappa)^{n-2}$$

$$= \frac{n+\kappa}{n}(1+\kappa)^{n-2},$$

and therefore $\overline{u_1^2} = \overline{u_2^2} = \&c.$

$$= \frac{1}{h\mathrm{M}}\frac{\mathrm{D}_{11}}{\mathrm{D}} = \frac{1}{h\mathrm{M}}\cdot\frac{1}{n}\frac{n+\kappa}{1+\kappa}.$$

And since v^2 and w^2 have corresponding values, therefore

$$\overline{n\mathrm{T}} = \mathrm{M}\frac{3n}{2h}\overline{u_1^2} = \frac{3}{2h}\cdot\frac{n+\kappa}{1+\kappa}.$$

These results are easily obtained by considering the general determinant.

$$\mathrm{D} = \begin{vmatrix} 2a & b & b & \cdots \\ b & 2a & b & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{vmatrix}$$

It will be found for $n=2$, $n=3$, and thence by induction for all values of n, that $\mathrm{D} = (2a-b)^n + nb(2a-b)^{n-1}$. Whence, replacing $2a$ by $1+\frac{n-1}{n}\kappa$, and b by $-\frac{\kappa}{n}$, we get the results above stated.

Again we find

$$\overline{n\mathrm{T}_r} = \frac{3}{2h}\frac{n-1}{1+\kappa},$$

and

$$\overline{n\mathrm{T}_s} = \overline{n\mathrm{T}} - \overline{n\mathrm{T}_r} = \frac{3}{2h},$$

and therefore $\frac{\mathrm{T}_s}{\mathrm{T}} = \frac{1+\kappa}{n+\kappa}$, which increases as κ increases, that is *ceteris paribus* as the diameter of the spheres increases.

Again the mean pressure per unit of area is $p = \frac{2}{3}(1+\kappa)\rho\overline{\mathrm{T}_r}$, which is independent of c. For a system of material points $p = \frac{2}{3}\rho\mathrm{T}_r$, that is $\frac{2}{3}(1+\kappa)\rho\mathrm{T}_r$, since in this case $\kappa = 0$. As the spheres increase in diameter with $(1+\kappa)\mathrm{T}_r$ constant, p remains constant.

The number of collisions per unit of volume and time varies as $c^2\sqrt{\mathrm{T}_r}$, that is, as $\frac{\kappa^{\frac{2}{3}}}{\sqrt{1+\kappa}}$, and is, therefore, less than it would be if, with the same diameter, the spheres had velocities independent of each other.

20. It follows from the fact that p is independent of κ, that local variations of density, that is of κ, involve, on the whole, no expenditure of work, and will, in fact, come into being.

21. The effect of collisions between the spheres is now considered directly, to show how we obtain the known results that collisions between members of a group of spheres tend to reduce the group to the "special state" in which T_r is constant throughout the group. Let the component velocities of two spheres be $x_1y_1z_1$ $x_2y_2z_2$ before collision and $x'_1y'_1z'_1$ $x'_2y'_2z'_2$ after collision. Then, if the two are members of a group and the chance that the members of the group shall have assigned velocities is $c\epsilon^{-h\mathrm{Q}}$, in which

$$\mathrm{Q} = ax_1^2 + bx_1x_2 + ax_2^2 + \&\mathrm{c}.,$$

the a coefficients being all alike and the b's all alike, we find that the chance for the velocities after the collision is $c\epsilon^{-h\mathrm{Q}'}$, in which Q' is the same function of $x'_1x'_2$, &c., that Q is of x_1x_2, &c. This shows that the distribution is not disturbed by collisions if all the a's are alike and all the b's alike. The group is in the special state.

22. But if the a's differ from each other or the b's differ from each other, it is shown that collisions tend to reduce them to equality, a with a and b with b; that is to reduce the group to the special state.

23. Boltzmann's minimum function tends to diminish by collisions, finally becoming constant for any group of contiguous spheres, when T_r becomes uniform throughout the group. On the other hand, as the group becomes too large, the spheres composing it develop an opposite tendency to split up into smaller groups, each with some

small stream motion relative to the others, and so to diminish the mean pressure and the number of collisions per unit of time. The actual state of the medium is a compromise between the two opposite tendencies.

Presents, February 7, 1895.

Transactions.

Basel:—Naturforschende Gesellschaft. Verhandlungen. Band X. Heft 2. 8vo. *Basel* 1894. The Society.

Batavia:—Bataviaasch Genootschap van Kunsten en Wetenschappen. Notulen van de Algemeene en Bestuursvergaderingen. Deel XXXI. Aflevering 3, en 4. Deel XXXII. Aflevering 1, en 2. 8vo. *Batavia* 1893–94; Catalogus der Ethnologische Verzameling. Vierde Druk. Supplement. 8vo. *Batavia* 1894; Nederlandsch-Indisch Plakaatboek 1602—1811. Twaalfde Deel, 1795—1799. 8vo. *Batavia* 1894; Tijdschrift voor Indische Taal, Land en Volkenkunde. Deel XXXVII. Aflevering 1—5. Deel XXXVIII. Aflevering 1 en 2. 8vo. *Batavia* 1893–94; Verhandelingen. Deel XLVII. 2^{e} Stuk. Deel XLVIII. 1ste Stuk. 8vo. *Batavia* 1893. The Academy.

Berlin:—Deutsche Chemische Gesellschaft. Berichte. 1894. Nos. 10—18. 8vo. *Berlin.* The Society.

Brussels:—Académie Royale de Médecine. Bulletin. Tome VIII. Nos. 5—10. 8vo. *Bruxelles* 1894. The Academy.

Académie Royale des Sciences. Bulletin. Tome XXVII. Nos. 5—11. 8vo. *Bruxelles* 1894. The Academy.

Florence:—Biblioteca Nazionale Centrale. Bollettino. 1894. Giugno-Dicembre. 8vo. *Firenze.* The Library.

London:—Chemical Society. Journal. July—December, 1894. 8vo. *London.* Proceedings. Nos. 140—146. 8vo. *London* 1894. The Society.

Geological Society. Abstract of the Proceedings. Nos. 628—634. 8vo. *London* 1894; Quarterly Journal. Vol. LI. Part I. 8vo. *London* 1895. The Society.

Institution of Civil Engineers. Abstract of Proceedings. Session 1894–95. Nos. 1—4. 8vo. *London.* The Institution.

Institution of Electrical Engineers. Journal. Vol. XXIII. Nos. 112, 113. 8vo. *London* 1894. The Institution.

Pharmaceutical Society of Great Britain. Pharmaceutical Journal and Transactions. July—December, 1894. 8vo. *London.* The Society.